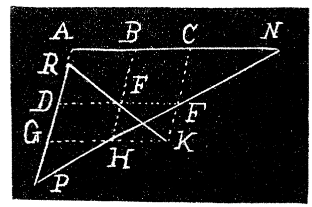
Solution by G. I. HOPKINS, A. M., Professor of Mathematics and Astronomy, Manchester (N. H.) High School, and MISS IDA M. SCHOTTENFELS, A. M., New York, N. Y.

Let A, B, C, D, E, F, G, H, and K be the points. It is evident that A, C, K, and G will be the vertices of a parallelogram. Let BH be a median of this parallelogram, and E any point in the median except the center. Then the broken line $ANPRK$ will fulfill the conditions of the problem. This course fails if the middle row and middle column bisect each other. If the row DEF is not parallel to GK, then three lines, or a broken line of three segments will fulfill the conditions of the problem.



G. I. HOPKINS.

If BH and DF are not medians, take the course KER, RP, PN, NB. If BH and DF are medians, take the course KEA, AN, NP, PD; or KEA, AP, PN, NB.

IDA M. SCHOTTENFELS.

Also solved by the Proposer.

296 (Incorrectly numbered 294). Proposed by JOHN JAMES QUINN, Ph. D., Scottdale, Pa.

a) Suppose an indefinite line be pivoted at the end of a revolving radius whose center is the origin; and the initial position of the radius is coincident with the X-axis and the pivoted line perpendicular to it. As the radius revolves through equal amounts of arc the line moves to the right over corresponding equal intercepts on the X-axis. What is the equation of the locus of a point on the line whose distance from the end of the radius is equal to a diameter?

b) Show how the locus can be applied to the multisection of an angle.

c) Suppose the diameter be laid off in both directions.

No solution of this problem has been received.

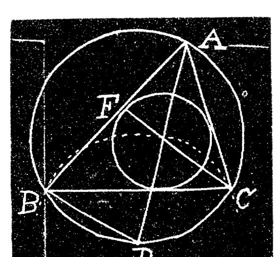
297 (Incorrectly numbered 295). Proposed by S. F. NORRIS, Professor of Mathematics, Baltimore City College, Md.

One side and the opposite angle of a triangle are fixed. Find the locus of the center of the inscribed circle. Solve by methods of analytic geometry.

I. Solution by C. N. SCHMALL, A. B., 89 Columbia Street, New York City.

This problem can be solved more easily and more neatly by Euclidean Geometry. Thus, referring to figure, we have,

$$\angle DCO = \angle DCB + \angle BCO = \angle DCB + \tfrac{1}{2}\angle BCA,$$
$$\angle DOC = \angle AOF = \angle OAC + \angle OCA;$$



but $\angle OAC = \angle DCB$ (since arc $DC=DB$), and $\angle OCA = \frac{1}{2}\angle BCA$. Hence $\angle DCO = \angle DOC$.

Therefore $DC=DO$.

Hence, keeping BC constant and vertex A always on the arc BAC (making opposite angle constant) the locus of center O of the inscribed circle is a circle whose center is D and radius DC.

II. **Solution by G. W. GREENWOOD, A. M., Dunbar, Pa.**

Taking as x-axis the fixed side AB $(=2a)$ and its mid-point as origin, the in-center, since it lies on the bisectors of the angles A and B, must satisfy the equatiōns $y=m_1(x-a)$, $y=m_2(x+a)$, where

$$m_1=\tan\left(\pi-\frac{A}{2}\right)=-\tan\frac{A}{2},\quad m_2=\tan\frac{B}{2}.$$

The opposite angle being constant, $A+B$ is constant. Hence

$$\tan\left(\frac{A}{2}+\frac{B}{2}\right)=\frac{-m_1+m_2}{1+m_1m_2}=\text{a constant}=c,\text{ say.}$$

Hence the in-center satisfies the equation $c(x^2+y^2-a^2)+2ay=0$.

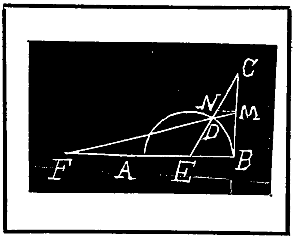
Also solved by J. Scheffer, William Hoover, and L. E. Newcomb.

298 (Incorrectly numbered 296). Proposed by J. J. QUINN, Ph. D., Scottdale, Pa.

Given $AB=BC$ perpendicular to each other, and E and M their mid-points, respectively. On AB describe a semi-circle, and draw CE to meet the circumference in D. Draw DM cutting AB in F. In what ratio is AB divided by the point F?

Solution by C. N. SCHMALL, A. B., 89 Columbia Street, New York; L. E. NEWCOMB, Los Gatos, California; and A. H. HOLMES, Brunswick, Maine.

From the figure, constructed as described in the problem, we have



$FE:ED=NM:ND$. But $NM=\frac{1}{2}EB$, and

$$ND=CD-CN=CD-\tfrac{1}{2}CE\ldots(1).$$
$$\therefore FE:ED=\tfrac{1}{2}EB:CD-\tfrac{1}{2}CE\ldots(2).$$

Also $AB^2=CD.CD'$, where D' is the second intersection of the secant, CE, with the circle. Now let $FA=x$, $EB=r$, $CD=a$. Then $FE=x+r$, $CE=a+r$. Substituting these values in (1) and (2), we have

$$x+r:r=\tfrac{1}{2}r:a-\tfrac{1}{2}(a+r)\ldots(3),$$
$$(2r)^2=a(a+2r)\ldots(4).$$

From (4), $a=r(\pm\sqrt{5}-1)$, and from (3),

$$a=r+\frac{r^2}{x+r}.\quad \therefore r(\pm\sqrt{4}-1)=r+\frac{r^2}{x+r}.$$

Whence $x=(\pm\sqrt{5}+1)r$. $\therefore FA/AB=\frac{1}{2}(\pm\sqrt{5}+1)$.

Also solved by G. W. Greenwood and J. Scheffer.